LESSON 2
AN 8-SECOND TRIP

NCTM Mathematics Standards

Algebra

Measurement

Geometry

Statistics

Mission

To determine the speed of various toys by conducting experiments

To calculate the speed of various means of transportation by using algebra

With car chases and thrilling airplane maneuvers in action-based movies, students have witnessed many events involving speed. In spite of this exposure, students have little experience trying to quantify speed and compare rates of speed. This lesson provides several experiments involving familiar objects from the students' world with which they explore the concept of speed. These hands-on experiences are used as a basis for increasing students' understanding of the speed of conventional modes of transportation.

The Speed Demon rocket car can attain speeds of up to 300 miles per hour. (Photo courtesy of Brent McNeely, Hell on Wheels, Inc.)

PURPOSE

The goal of teaching mathematics is to help students gain mathematical power. They need experiences using mathematical reasoning, solving problems, and making connections to the real world. In this lesson, students explore the concept of speed as it relates to travel in space. The mathematics of this topic is rich. Students use algebra, measurement, geometry, and statistics to improve their understanding of speed.

Part A

GETTING STARTED

NCTM Teaching Standards:

Teachers of mathematics should pose tasks that are based on—

- ***sound and significant mathematics;***
- ***knowledge of students' understandings, interests, and experiences;***
- ***knowledge of the range of ways that students learn mathematics.***

(NCTM 1991, p. 25)

In lesson 1 of this module, students learned how far they could move in 8 seconds. During the opening discussion, some students may have talked about traveling at high speeds in cars and airplanes. From students' suggestions, list ways to travel fast. This list will probably contain several types of automobiles and airplanes as well as other vehicles. Consolidate the list by considering only large categories, such as bicycles, motorcycles, cars, airplanes, and jet airplanes. The Space Shuttle can be a category in itself.

Continue the discussion by asking students to recommend some miniature test vehicles, since the speed of jet airplanes and cars cannot be tested in the classroom or around the school.

Usually every classroom contains a few students who collect small toy cars that come with ramps and loops. Other students may own battery-powered remote-control cars. Some may have experimented with small toy rockets. Some or all of these vehicles can be used in and around school to help students collect data about speed.

Teachers of mathematics should promote classroom discourse in which students use a variety of tools to reason, make connections, solve problems, and communicate.

(NCTM 1991, p. 45)

DEVELOPING THE ACTIVITY

This lesson is most effective if students have an opportunity to collect and organize data about the speed of familiar objects. This experience will be valuable before they consider automobiles, airplanes, and other means of transportation from the adult world. Specific directions for toy cars, remote-control cars, and small toy rockets follow. Do as many of these activities as possible.

Toy Cars

Toy cars rolling down a ramp may not remain in motion for 8 seconds. Students need to time the cars and calculate the speed of the cars using the formula $d = rt$, where d = the distance traveled, r = the average speed of the car, and t = the time traveled. The average rate of speed (r) can be calculated by manipulating the time-distance-rate formula algebraically.

Assemble approximately 3 feet of track. To keep the track straight, secure it to a board or strong yardstick. Elevate one end and roll a toy car down the track.

Although this lesson has "8 seconds" in the title, the stopping time for a toy car is not likely to be exactly 8 seconds. This time can be affected by the ramp's surface and other variables. Before having them break into groups, let students make a few trial runs to determine a reasonable time for the car to come to a stop. All groups should use the same time interval, for example, 5 seconds. It becomes a constant in the data.

Group the students into mission teams of four students for the collection of data. One student is the car starter, another is the recorder, and two are timers. They can rotate tasks for different trials and ramps.

Have each mission team describe its ramp with the ratio of the height of the ramp to the horizontal length of the ramp. Have students collect three to five sets of data about the car and any given ramp. They will be measuring the distance traveled over the agreed-on constant time.

Students may think that they can construct a better ramp. Allow them to make some adjustments and collect more data. To ensure comparable data throughout the class, require three ramps from each group. Each mission team should collect data on a ramp with a 1/3, 1/4, 1/5, and 1/6 ratio. Be certain that students release the cars from the same point on the ramps during the trials. It is best to mark the release point with a piece of tape to assure that this occurs. It is appropriate to have each group predict which ramp would produce the highest speed.

The data should be organized as shown below, and the average speed should be calculated in feet per second.

Sample Data
Mission Team: Green

Ramp Ratio	Trial	Distance (ft)	Time (s)	Avg. Rate of Speed (ft/s)
1/3	1	8	4	2
1/3	2	8	5	1.6

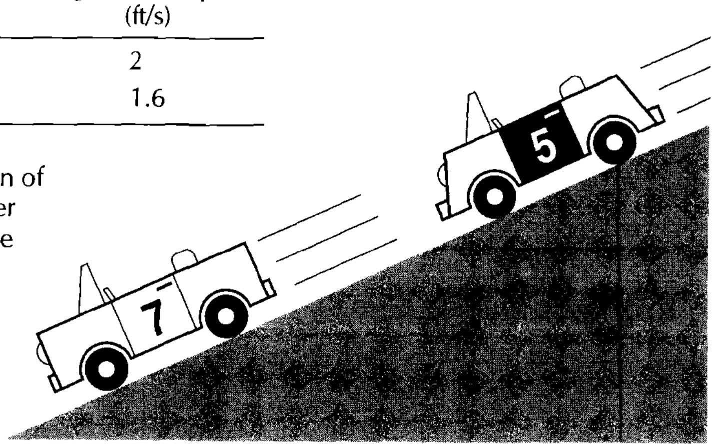
Students are accustomed to miles per hour as a description of speed. Some may want to convert the results into miles per hour because that is the only everyday reference they have for speed. Help them convert a slow speed, say, 20 miles per hour, to feet per second so they have a comparison.

To close this particular experiment, have students compare data about the ramps and the speed of the cars. Have them compare their results with the earlier predictions.

Materials for toy cars: ***At least one toy car, at least 3 feet of track for a ramp, a board or strong yardstick, another yardstick or tape measure, masking tape, and stopwatches***

Teaching Note: ***In reality, a car rolling down a ramp constantly accelerates, so the speed increases as the car nears the bottom of the ramp. The behavior of the car as it accelerates down the ramp and its behavior as it continues across the table or floor are different. Ideally, the car would travel across the table or floor at constant velocity, but its speed actually is retarded by friction. To simplify this activity for middle school students, we are determining the average rate of speed of the car.***

Teaching Tip: ***If students have reached a point in the curriculum where it is appropriate to use the Pythagorean theorem, allow them to calculate the lengths of the legs of the triangle formed by the ramp.***



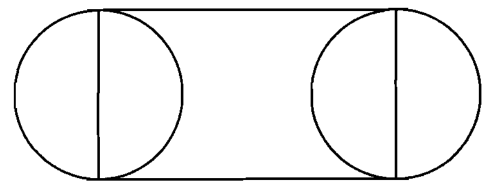
Toy cars on a ramp

Battery-Powered Remote-Control Cars

Materials for battery-powered remote-control cars: ***One or more cars, stopwatches, yardsticks or measuring tape, masking tape, "traffic" cones from the physical education teacher, and extra batteries***

At least one battery-powered remote-control car is needed for this experiment. These cars come in many varieties: trucks, stock-car racers, dragsters, and so on. It is recommended that the first test of speed be done on an oval or circular track of known length. These cars can be very quick. On a straight track, the car can go quite a distance in 5 to 8 seconds. The closed track can be constructed indoors in a gymnasium, in a cafeteria, or on a stage as well as outside in the schoolyard.

Describe a track made of a rectangle with a semicircle on each end. In mission teams, the students design their own track. The students' experiences with formulas for perimeter and circumference may determine the steps they take in designing the track. Some students might develop a track shape, then measure the distance.



Alternatively, they can select a track length and then design a track with that distance. A circular track is acceptable and can be "drawn" by students using a length of string with a piece of chalk tied to the end as the radius. The track designs from all the groups should be displayed and discussed. For the first trial, the teacher should select an appropriate track for the available space. If more than one design is usable, set up more tracks to collect data. Define the track by placing traffic cones inside the track at such points as the beginning of turns and straightaways.

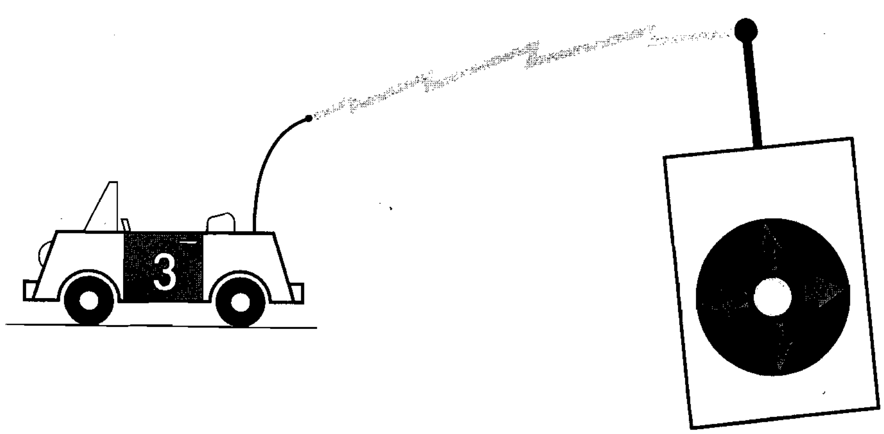
NCTM Teaching Standards: ***The teacher of mathematics should create a learning environment that fosters the development of students' mathematical power by using the physical space in a way that facilitates students' learning of mathematics.***
(NCTM 1991, p. 57)

To conduct the experiment, each mission team should collect data on the time required to complete one lap around the track. This time will be used to compute the average speed of the car in one lap. Members of the mission teams can play the roles of starter, timer, recorder, and driver. Each student may want to "drive." Four trials for each team may be necessary to involve all students.

The data from the experiment should be collected in a table by each mission team, as illustrated.

Sample Data
Mission Team: Blue

Track	Driver	Distance (ft)	Time (s)	Avg. Rate of Speed (ft/s)
1	Sue	54	6	9
1	Tina	54	18 (spin out)	3



A remote-control car

The closing of this activity is interesting. Because the data will vary greatly from one trial to another, a class discussion will be necessary about how to report central-tendency data from the mission teams to the class. What approach is most advantageous if the teams are competing? What result can be used to be the most consistent across teams? What data best describe the speed of the battery-powered cars? Regardless of how the data are reported, students are gaining an intuitive and formal understanding of speed through these experiments.

Toy Rockets

All rockets must be flown outdoors. The teacher must plan carefully to ensure that all safety precautions are followed. This is a time to discuss the importance that NASA places on safety and to assure the class that the same consideration will be applied in this experiment. All forms of rockets are sold with very specific directions related to construction and safety concerns. Read these directions carefully and make students aware of their importance.

Toy rockets are available in toy stores in several forms: compressed-air, water-powered, and solid-fuel rockets. It is likely that some students will own these or will have had experience flying them in a hobby club. The discussions here are based on solid-fuel rockets, since they require the most preparation and involve significant safety concerns.

The speed of rockets is more difficult to measure than the speed of toy cars because rockets are traveling vertically. Accurately measuring the distance traveled is not possible with the tools at our disposal. Most students in middle school are not ready to measure the height of an object indirectly. For the majority of students, benchmark heights, such as the school building, a tree, or the flag pole, help them estimate the height of the rocket flight. However, if this experiment is done with algebra students in middle school, they can understand the basic trigonometry necessary for indirect measurement.

In some classes, students can bring in rockets from home. In other classes, no one has rockets to share. When rockets and related equipment must be purchased, the cost prohibits each student from receiving a rocket. This experiment then becomes a whole-class activity.

Student and parent volunteers can begin building the rockets and associated equipment about two weeks before the anticipated data-collection event. This phase needs to be done only for the first school year in which the experiment is done. In later years, only replacing the fuel will be required. For the data-collection event, group students into their mission teams. All teams can collect data simultaneously. Two mission team members watch and estimate the height of the rocket flight, and two measure the time. Even with a warning about the short flights, students miss the start or forget to push the stop button on the stopwatch when the rocket reaches its highest point. Usually, one of the two students gets both tasks done. Since the time duration is very short, variance occurs in the measures of the time even if all students start and stop their watches appropriately. The jobs can rotate among team members from trial to trial. To get reasonable data, at least three rockets should be fired in each class. For both the height and the time, the two team members must agree on a value for the data chart.

Sample Data

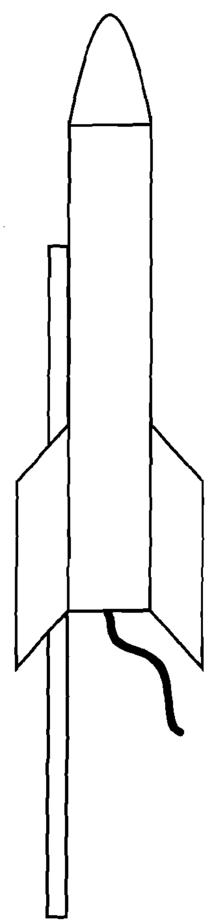
Mission Team: Red

Rocket	Trial	Distance (ft)	Time (s)	Avg. Rate of Speed (ft/s)
1	1	60	1.5	40
2	1	75	2	37.5

Among the mission teams, data usually vary. Discuss as a whole class how to summarize the data. Make a line plot of data for each rocket and trial. Discuss how to handle data that seem to be far from the others and how to use central tendency to describe those data that seem consistent. Try to come to a class agreement on an estimate for the average speed of these rockets in feet per second. Teachers may want to to refer to *Mission Mathematics: Grades K–6* for another rocket activity, "Fizzy-Tablet Rockets." In that activity, antacid tablets are used as fuel to launch film-canister rockets. The duration of these flights is short, but it is possible to use this alternative activity to measure speed.

NCTM Teaching Standards: ***The teacher of mathematics should orchestrate discourse by deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let students struggle with a difficulty.***
(NCTM 1991, p. 35)

Materials for toy rockets: ***At least three rockets for each class, one rocket launch pad, a 12-volt battery, and stopwatches. Clinometers or hypsometers would be necessary to make indirect measurements using basic trigonometry.***



A toy rocket

CLOSING THE ACTIVITY

The closing of this part of the lesson varies somewhat, depending on which experiments are completed. However, each experiment indicates that summarizing the data should be part of the closing. Students should include these summaries in their log books along with accounts of the mathematics they used and learned in the experiments. Of course, they should record what they learned about speed from their experiments with toys. They should begin thinking about the speeds of running animals, swimming creatures, and various vehicles. Asking them to write two sentences about speed in everyday life will give insight into their present concept of speed.

Students can continue developing ideas for their mission patches.

Part B

GETTING STARTED

NCTM Teaching Standards: ***The teacher of mathematics, to enhance discourse, should encourage and accept the use of computers, calculators, and other technology.***

(NCTM 1991, p. 52)

In lesson 1, students collected data about their speed for 100 feet. Earlier in this lesson, they collected data about the speed of selected toys. These data can be entered on a class chart and discussed. The data table included on data sheet 1 on page 66 contains some data about the top speeds of some vehicles. If more data are desired about actual speed records, the *Guinness Book of World Records* (1996) may be a useful reference. Students are invited to compare the two tables. Since one is in feet per second and the other shows miles and hours, comparing the data is problematic.

DEVELOPING THE ACTIVITY

40 ft/s × 60 s/min = 2 400 ft/min

2 400 ft/min × 60 min/h = 144 000 ft/h

144 000 ft/h × 1 mi/5 280 ft = 27.27 mi/h

The problem at hand is how to convert the units in the data tables to allow comparisons. Since most of the earlier data are in feet per second, encourage students to convert the data on sheet 1 to those units.

This is a wonderful opportunity to use technology to help solve the problem. Perhaps the most appropriate technology is a spreadsheet. If the class can go to a computer laboratory where students can work independently or in pairs, the spreadsheet experience can be valuable in getting them to think algebraically.

The spreadsheet should have the following columns: Mode of Travel, Distance, Time, Distance in Feet, Time in Seconds, Speed in Feet per Second, and Distance in 8 Seconds. Students can enter the data in the first three columns from the data table. Through questioning, lead the class to propose how to convert miles to feet. Show them how to enter the multiplication of a constant, 5 280, and each number in the Distance column by using a formula. Additionally, they can copy and paste the formula throughout the Distance in Feet column.

The discussion of converting the time data to seconds involves a more complex formula. It may be appropriate to include exponents in the formula if this topic has been covered in the curriculum. Whether exponents or multiplying by 60 twice is used in the formula, both hours and minutes must be converted to seconds. Since this formula is more complex, it is most appropriate to show students how the spreadsheet's copy command is used.

This entire conversion can be accomplished without computers. The calculations can be done by using calculators or paper and pencil.

With the data table complete, students are challenged to graph the results. Expect students to find some difficulty making the graph. Because of the speed of the Space Shuttle, the range of the data is very large. The choice of a unit becomes difficult for a clumped group of data and one extreme outlying piece of data. One solution to this problem is to omit the Shuttle's speed and graph the remaining data. Look at the graph and discuss what it would look like if the data were transformed into miles per hour.

CLOSING THE ACTIVITY

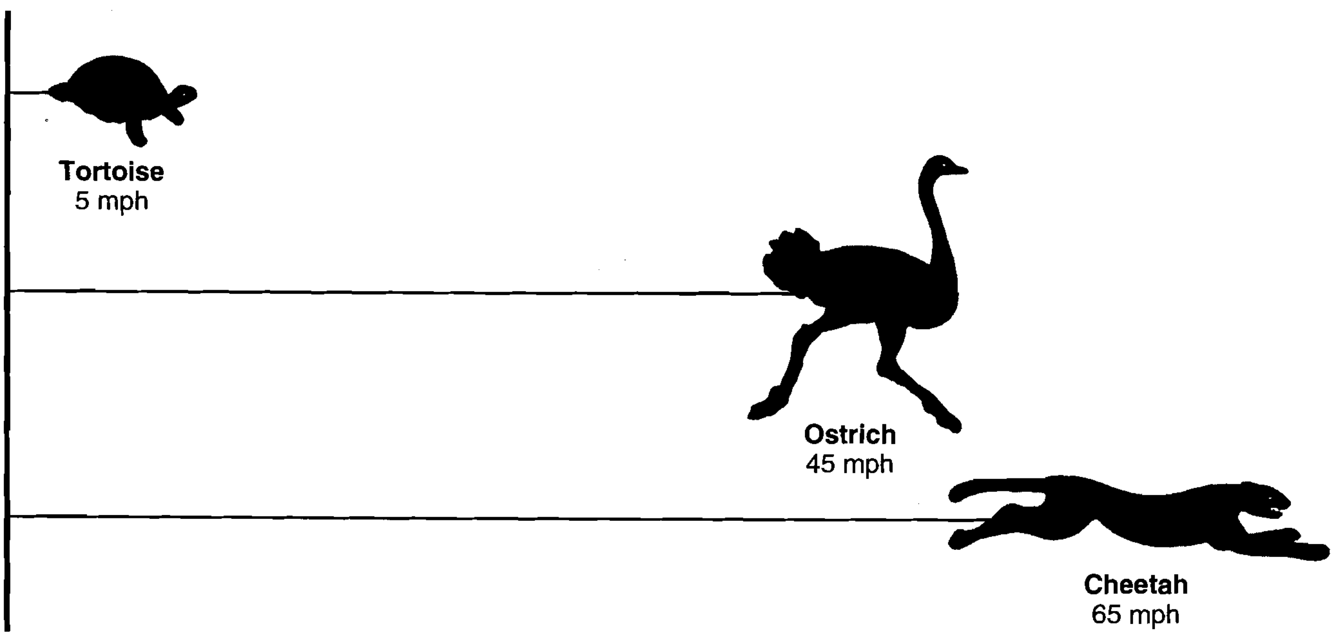
Log-book writing is an important part of the lesson. First, students should write about the mathematics they have encountered and, perhaps, the technology they have used in this lesson. Additionally, students should reflect on the difficulty of putting the Shuttle's speed on the graph. On Earth, relatively little difference appears in the speed of vehicles. However, we see a quantum leap as we function in the realm of twentieth-century space travel. For humankind to travel to the outer planets and beyond, another quantum leap in our ability to travel in space will be needed.

NCTM Assessment Standards:
Assessments should enhance learning. They are learning opportunities as well as opportunities for students to show what they know and can do.
(NCTM 1995, p. 13)

Students should continue to develop their "8 Second" mission patches if time allows.

EXTENDING THE LESSON

Students are very often quite interested in the notion of speed, and this lesson only stimulates this interest to a higher level. Another approach to the discussion is to use the maximum speed of animals. Crawling insects, mammals of the sea, birds, fish, reptiles, and land-based mammals can all be used to develop charts of distance, time, and rate of speed. Hands-on activities related to speed can be accomplished using hamsters and turtles in classrooms. Animals can be released at the center of a circle and timed on their exit from the circle. Also, the class could have a "mutt derby" for their pet dogs. On a straight track, students can record the time for animals to complete a short race and determine the average speed of their pets.



Top speed for various animals